Plug-and-Play Model Predictive Control for Water Supply Networks with Storage

Kallesøe, Carsten Skovmose; Jensen, Tom Nørgaard; Bendtsen, Jan Dimon

Published in:
IFAC-PapersOnLine

DOI (link to publication from Publisher):
10.1016/j.ifacol.2017.08.616

Publication date:
2017

Document Version
Publisher's PDF, also known as Version of record

Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 50-1 (2017) 6582–6587

Plug-and-Play Model Predictive Control for Water Supply Networks with Storage

Carsten Skovmose Kallesøe *,** **Tom Nørgaard Jensen** * **Jan Dimon Bendtsen** *

* *Aalborg University, Fredrik Bajers Vej 7c, DK-9220 Aalborg, Denmark (e-mail: {csk,tnj,dimon}@es.aau.dk)*
** *Grundfos, Poul Due Jensens Vej 7, DK-8850 Bjerringbro, Denmark (e-mail: ckallesoe@grundfos.com)*

Abstract: This paper addresses cost optimal control of pumping stations in water distribution networks with elevated reservoirs. We consider a topology in which a single pumping station and a single elevated reservoir is present in the network. This configuration is often seen at smaller water utilities. Typically, advanced network models and staff with control experience are not available at such utilities, therefore we pursue a plug-and-play approach that identifies a reduced network model from measurements and use the obtained model in an economic model predictive control (economic MPC) scheme.

Keywords: Water Distribution, Model Predictive Control, Parameter Identification, Model Reduction, Smart Grid.

1. INTRODUCTION

This paper deals with optimization of reservoir filling via pump control in water distribution networks containing one pumping station and one elevated reservoir. Optimality is here measured as the minimum cost for operating the pump station. This cost is a function of the power consumption of the pumping station and fluctuating energy prices. The aim is to develop a plug-and-play solution enabling the use of the method at small water utilities where the development of complex algorithms and detailed network models are economically out of reach. The configuration with one pumping station and one elevated reservoir is often used at small utilities, as pump control becomes particularly simple in this case. The pumping station is connected to the reservoir through a main pipeline, and, in addition to the reservoir, a number of small town districts are also connected to the main pipeline, see Fig. 1.

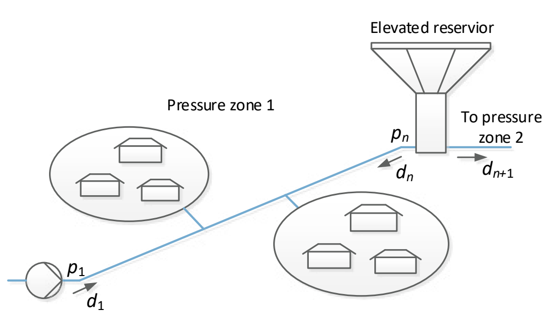

Fig. 1. Sketch of the type of water distribution network considered in this paper. The control objective is to control the flow d_1 to minimize the operating cost of the pump with variable energy prices taken into consideration.

Optimal control of pumping stations in water distribution is investigated in many papers. Ocampo-Martinez et al. (2013) describes a Model Predictive Control (MPC) framework for controlling both water distribution and wastewater networks. Distributed MPC control of water distribution networks is considered in Leirens et al. (2010), and a decentralised MPC approach is proposed in Ocampo-Martinez et al. (2011). Sun et al. (2014) considers the non-linearities of the hydraulic network using a constraint satisfaction problem formulation.

The MPC control scheme proposed in the current exposition utilizes a reduced network model, which is also the main contribution. Reduced network models are also treated in Kallesøe et al. (2015), Maschler and Savic (1999), Paluszczyszyn et al. (2013). In Kallesøe et al. (2015) a reduced network model that is identified from measurements is used for pumping station control. While the networks investigated in Kallesøe et al. (2015) are without elevated reservoirs, we here consider networks including reservoirs. Maschler and Savic (1999) proposes model reduction for water distribution network models by using knowledge about the underlying graph. Paluszczyszyn et al. (2013) proposes a fast algorithm for reducing network models for various control and network analysis purposes. This algorithm is also based on the underlying graph of the network. While we in the current exposition assume the underlying graph of the network to have a tree structure as the system shown in Fig. 1, we have no assumptions about the distribution of the demand nodes in the tree. Nevertheless, the reduced model, which we propose, is able to capture the relation between flow and pressure to a degree which is sufficient for the purpose of our control scheme, while having the benefit of low complexity and few parameters.

Methods for online identification of user demands as well as identification of the parameters of the reduced network

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2017.08.616

model are proposed in the paper. After identification of the parameters of the reduced model, it is subsequently utilized in a plug-and-play control approach Stoustrup (2009) for economic Model Predictive Control (economic MPC). Referring to the control structure presented in Ocampo-Martinez et al. (2013), the proposed economic MPC is placed on the Control Level, while a local pump controller is assumed at the Local Control Level. The resulting control scheme is self commissioning and adaptive (plug-and-play). Thereby the approach presented here distinguishes from Baunsgaard et al. (2016) by the structure of the network under consideration and the automatic identification of the user demands and the network model parameters.

The paper starts by presenting a reduced network model of the network sketched in Fig. 1. This is described in Section 2. In Section 2, the developed model is utilized in a parameter identification scheme that can identify the parameters of the model in Section 3. In Section 4, the model is utilized in a nonlinear MPC scheme. Section 5 presents the results of a numerical experiment which exemplifies the operation and benefits of the proposed control scheme. The paper ends with some concluding remarks.

Nomenclature: We use boldface low case letters for vectors and boldface capital letters for matrices such that $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{A} \in \mathbb{R}^{n\times m}$. $\mathbf{1}$ indicates a vector with one on all entries. By $\mathbf{A} > (\geq)0$ we denote positive (semi-) definiteness of the matrix $\mathbf{A}$. For a vector $\mathbf{x} \in \mathbb{R}^n$, we use $\mathbf{x} > (\geq)0$ to denote $x_i > (\geq)0$ for every $i \in \{1, 2, ..., n\}$. $\mathbb{1}_{x>0}$ is an indicator function such that $\mathbb{1}_{x>0}$ equals 1 when $x > 0$ otherwise it is 0.

2. REDUCED SYSTEM MODEL

The network in Fig. 1 is divided into several districts, collectively referred to as “Pressure Zone” (PZ). The flow from the pumping station into PZ1 is denoted d_1 and is the control variable in this work. The flow from the reservoir into PZ1 is denoted d_n and can be both positive and negative. Finally, the reservoir also supplies other parts (PZ2) of the network. The collective flow to these parts is denoted d_{n+1}, which is always positive.

The pressure at node n that connects the reservoir to PZ1 is a function of the level h in the reservoir. This level is a function of the flow leaving the reservoir (d_n and d_{n+1}) and is described by

$$A\dot{h} = -d_n - d_{n+1} \tag{1}$$

where A is the constant cross sectional area of the reservoir and h belong to an interval restricted by the height of the reservoir. The pressure at the connecting node is found from the level by

$$p_n = \alpha h + \alpha h_0 \tag{2}$$

where p_n is the pressure at node n, α is the scaling between the water level and the pressure unit and h_0 is the geodesic offset between node n and the geodesic level where $h = 0$ (zero point of the level sensor). Typically, the level is measured in meters and the pressure is measured in bar, hence α scales from meter to bar.

The algorithm developed in this paper is restricted to networks with the structure shown in Fig. 1. This means that PZ1 is well modelled by a simple tree graph where each district is described as demand flow of the corresponding connection node (Maschler and Savic (1999)), see Fig. 2.

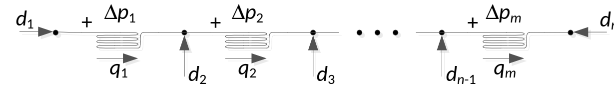

Fig. 2. Simple tree graph that models the relation between the pressure at node 1 and the pressure at node n as a function of the demand flows d_1 to d_n.

Because of mass conservation in the network, the relation between the supply flow d_1, the reservoir flow d_n and the remaining demand flows is $d_1 + d_n = -\sum_{i=2}^{n-1} d_i$. The sign convention of the demand flow is such that $d_i \leq 0$, meaning that water is always consumed inside the district. Under the assumption that the consumers of the districts has similar behaviours in average, then the demand flow d_i to the individual districts is

$$d_i = -v_i\bar{d} \qquad \forall\, i = 2, \cdots, n-1 \tag{3}$$

where $\bar{d} = d_1 + d_n$ is the sum of demand flows, v_i describes the distribution of the district demands and is constant for all i, and $\sum_{i=2}^{n-1} v_i = 1$ due to mass conservation. The assumption that v_i is constant for all i implies that all district demands have the same daily profile. The study in Aquacraft (2011) implies that the assumption holds between districts with an equal distribution between consumer types (i.e., the distribution between residential-, industrial-, and other types of end-users is the same in all districts composing the PZ).

Now, define $v_1 = 0$, then the evaluation of the conservation of mass at the nodes of the graph in Fig. 2 implies that the pipe flows q_i are

$$q_i = d_1 - \bar{d}\sum_{j=1}^{i} v_j \quad \forall i = 1, \cdots, m \tag{4}$$

where m is the number of pipe elements, see Fig. 2. Note that $m = n - 1$ since the underlying graph of the network is a tree. The pressure Δp_i across each of the pipes in the graph is described by $\Delta p_i = r_i|q_i|q_i + \alpha h_i$, where q_i is the flow through the pipe, r_i is the pipe resistance, and h_i is the level change from the inlet to the outlet node of the pipe.

Using (4), and the fact the the sum of the pressure drops between p_1 to p_n equals $\sum_{i=1}^{m} \Delta p_i$ we have

$$p_1 = \sum_{i=1}^{m} r_i \left| d_1 - \sum_{j=1}^{i} v_j\bar{d} \right| \left(d_1 - \sum_{j=1}^{i} v_j\bar{d} \right) + \alpha\sum_{i=1}^{m} h_i + p_n \tag{5}$$

This model shows that the pressure at the supply node 1 is a function of the network structure (number of demand nodes), the distribution of the demands between the individual districts v_i, and the pipe resistances r_i.

The plug-and-play approach we pursue in this paper requires automatic identification of the model (5). We do this by defining a heuristic reduced network model that approximates (5) for control purposes. The structure of the reduced network model is

$$\begin{aligned} p_1 =& \theta_1|d_1|d_1 + \theta_2|d_1 - \bar{d}|(d_1 - \bar{d}) + \\ & \theta_3|d_1|(d_1 - \bar{d}) + \theta_4|d_1 - \bar{d}|d_1 + \theta_5 + p_n \end{aligned} \tag{6}$$

where θ_1 to θ_5 are chosen to minimize the distance between the reduced model (6) and the real model (5). The reduced model fits the real model perfectly when $m = 2$ with $\theta_1 = r_1$, $\theta_2 = r_2$, $\theta_3 = \theta_4 = 0$, and $\theta_5 = \alpha\sum_{i=1}^{m} h_i$. In

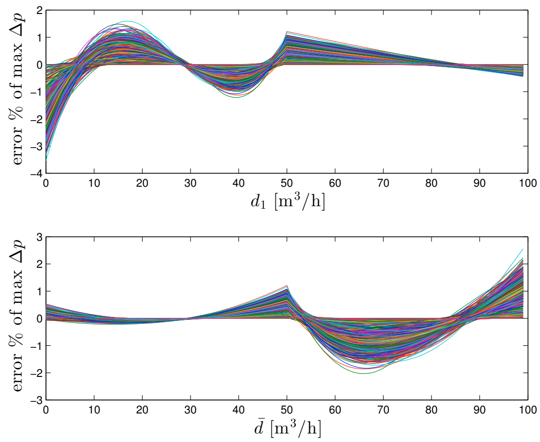

Fig. 3. Estimation errors for 2000 random tests on the network structure in Fig. 2. Here, Δp is the pressure across the pipeline and is given by $\Delta p = p_1 - p_n$.

the case where $m > 2$ (6) is only an approximation.

The model accuracy is tested with a random analysis where the number of nodes are chosen randomly between 1 and 50 and the distribution between the demand flow v is chosen randomly. The resistances are chosen randomly, with the restriction that the pressure difference Δp between node 1 and n is close to 1 bar at maximum supply flow which is 100 m^3/h, as this pressure drop is typical in the type of network considered here. The error between the actual and the estimated pressure is plotted in Fig. 3. The error is in percentage of the maximum pressure experienced in the test. The test shows that the estimation error is always below 4%, which we consider to be acceptable.

The numerical analysis presented in Fig. 3 shows that for this simple network structure and the restriction to 1 bar pressure difference between nodes 1 and n the approximation is good and can be used in the plug-and-play control approach proposed in Section 4.

3. SYSTEM IDENTIFICATION

Section 2 presented a reduced network model (6) that predicts the pressure at node 1 given the pressure at node n. This model is a good approximation under the assumptions that the network of PZ1 has a structure similar to the network shown in Fig. 2 and that the distribution between the user demands of the district is fixed under varying loads $\bar{d}$ (v_i constant for all i in (3)).

The dynamics of the level in the reservoir, shown in Fig. 1, is described by (1), and the level relates to the node pressure at node n in accordance with (2). This means that

$$\dot{p}_n = \frac{\alpha}{A} d_1 - \frac{\alpha}{A}(\bar{d} + d_{n+1}) \tag{7}$$

where the relation $d_n = -d_1 + \bar{d}$ is used. The total user demand $\bar{d}$ of the zone is a stochastic process with an inherent periodicity of one day (Alcocer-Yamanaka et al., 2012). The reservoir acts on the system as a low-pass filter filtering out fast variations in the flow, meaning that only the mean value of the daily demand is important when dealing with reservoir control. The daily demand variations differ between work days and weekends. In the following, we only consider a model for workdays but the model for the weekends and holidays is obtained in a similar way. Let $\frac{1}{A}\bar{d}(t) = g_{\boldsymbol{\lambda}}(t)$ where $g_{\boldsymbol{\lambda}}(t)$ is a periodic function given by

$$g_{\boldsymbol{\lambda}}(t) = \lambda_1 + \sum_{i=1}^{(k-1)/2} (\lambda_{2i}\cos(i\omega t) + \lambda_{2i+1}\sin(i\omega t)) \tag{8}$$

where ω is the angular frequency corresponding a period of one day and k is the number of frequency terms in the truncated Fourier Series. Other methods as for example a season model could have been used as well. However, the Fourier Series requires less data history, hence is well suited for implementation in distributed controllers.

Using a backward Euler approximation of the derivative $\dot{p}_n$ in (7) and the consumption demand model (8) the following relation is obtained

$$p_n(t) - p_n(t-\delta t) = \delta t \alpha \lambda_0 (d_1(t) - d_{n+1}(t)) - \delta t \alpha g_{\boldsymbol{\lambda}}(t) \tag{9}$$

where $\lambda_0 = \frac{1}{A}$. The expression (9) is linear in the parameters $(\lambda_0, \boldsymbol{\lambda})$ and therefore a good estimate of the parameters can be calculated, provided that $g_{\boldsymbol{\lambda}}$ can capture the shape of the consumption demand. Note that this can be controlled by increasing k in (8). Thus k should be chosen as a trade-off between model precision and model complexity.

We now assume that samples at times t_i $i = 1, \cdots, N$ of the pressure p_n and the flows d_1 and d_{n+1} are available for parameter identification, and let $t_i - t_{i-1} = \delta t_i$. The parameters of (9) are then found by solving the following optimization problem

$$\begin{aligned}\arg\min_{\lambda_0, \boldsymbol{\lambda}} \sum_{i=1}^{N} \big(p_n(t_i) - p_n(t_{i-1}) - \\ \delta t_i \alpha(\lambda_0(d_1(t_i) - d_{n+1}(t_i)) - g_{\boldsymbol{\lambda}}(t_i))\big)^2\end{aligned} \tag{10}$$

The optimization problem is linear in the parameters $\lambda_0, \boldsymbol{\lambda}$ meaning that standard recursive methods exist for the problem. Here, we use a recursive algorithm with a forgetting factor to enable tracking of changes in the system, Madsen (2008) and Ljung (1999). The recursive update is done once a day, meaning that a data-batch is collected over one day and used in the update.

To model the demand d_{n+1} of PZ2, we use a periodic function $g_{\boldsymbol{\mu}}(t)$. The arguments for $g_{\boldsymbol{\mu}}(t)$ being periodic are the same as for the demand in PZ1. The demand flow d_{n+1} is measured, therefore the optimization problem to identify the model parameters $\boldsymbol{\mu}$ of $g_{\boldsymbol{\mu}}$ is

$$\arg\min_{\boldsymbol{\mu}} \sum_{i=1}^{N} (d_{n+1}(t_i) - g_{\boldsymbol{\mu}}(t_i))^2 . \tag{11}$$

Equation (6) describes the relation between the supply pressure and -flow p_1, d_1, the reservoir pressure p_n, and the total user demand $\bar{d}$ in PZ1. The user demand $\bar{d}(t)$ is modelled by $Ag_{\boldsymbol{\lambda}}(t)$. Using the definition of λ_0 this means that $\bar{d}(t) = \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(t)$. Define the function $f_{\boldsymbol{\theta}}(d_1, t)$ such that

$$\begin{aligned} f_{\boldsymbol{\theta}}(d_1, t) = \theta_1 |d_1| d_1 + \theta_2 \left| d_1 - \frac{g_{\boldsymbol{\lambda}}(t)}{\lambda_0} \right| \left(d_1 - \frac{g_{\boldsymbol{\lambda}}(t)}{\lambda_0} \right) \\ + \theta_3 |d_1| \left(d_1 - \frac{g_{\boldsymbol{\lambda}}(t)}{\lambda_0} \right) + \theta_4 \left| d_1 - \frac{g_{\boldsymbol{\lambda}}(t)}{\lambda_0} \right| d_1 + \theta_5 \end{aligned} \tag{12}$$

then $p_1(t) - p_n(t) \approx f_{\boldsymbol{\theta}}(d_1, t)$. Assuming that samples at times t_i $i = 1, \cdots, N$ of the pressures p_1 and p_n, and the flow d_1 are available for parameter identification, then the parameters of $f_{\boldsymbol{\theta}}$ are found by solving the optimization problem

$$\arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} (p_1(t_i) - p_n(t_i) - f_{\boldsymbol{\theta}}(d_1(t_i), t_i))^2 \tag{13}$$

Again, as (13) is linear in the parameters ($\boldsymbol{\theta}$), recursive methods can be used to estimate the parameters.

4. PREDICTIVE CONTROL

Here, we propose a controller which minimizes the cost of operating the pumping station. The controller utilizes the elevated reservoir to move the power consumption in time, such that variable energy prices can be utilized to minimize the cost of operating the system. We assume that the energy price $c(t)$ is known in advance and is a function of time.

We further assume that the pumping station consists of a number of parallel connected pumps of equal size, and that a low level controller ensures optimal operation of the pumping station. That is, at a given flow and pressure, the number of pumps in operation is the number that results in the lowest power consumption. Under these operating conditions, the efficiency of the pumping station supplying a network can be assumed constant (Kallesøe et al., 2011) and the power consumption P is then $P = \frac{\Delta p_1 q_1}{\eta} = \frac{(p_1 - p_0)d_1}{\eta}$ with η the efficiency, p_0 the inlet pressure to the pumping station which is assumed constant, p_1 the outlet pressure which equals the pressure at node 1, and d_1 the supply flow which equals the demand flow into node 1.

The goal is to minimize the operation cost over a day by controlling the flow d_1. That is, we want to solve the minimization problem

$$\min_{d_1} \int_{t_0}^{t_0+T} 2c(\tau) \frac{(p_1(d_1(\tau), h(\tau), \tau) - p_0)d_1(\tau)}{\eta} \, d\tau + \kappa(h(t_0+T) - h(t_0))^2 \tag{14a}$$

where c is the cost and the term $(h(t_0+T) - h(t_0))^2$ is introduced because the system is periodic and therefore the start and end reservoir level over the period T should be the same. This minimization problem (14a) is constrained by the system dynamics

$$A\dot{h}(t) = d_1(t) - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(t) - g_{\boldsymbol{\mu}}(t) \tag{14b}$$

$$p_1(d_1, h, t) = f_{\boldsymbol{\theta}}(d_1, t) + \alpha h \tag{14c}$$

where $f_{\boldsymbol{\theta}}$ is the reduced network model described in (12) and $g_{\boldsymbol{\lambda}}$ and $g_{\boldsymbol{\mu}}$ are the prediction models for the user demands in PZ1 and 2. The constants λ_0 and α describing the size of reservoir and the scaling between level and pressure respectively. Constraints on the reservoir level and the supply flow are

$$0 \leq \underline{h} < h(t) < \overline{h} \quad , \quad 0 \leq \underline{d}_1 < d_1(t) < \overline{d}_1 \tag{14d}$$

Water quality is an important parameter when controlling water networks with reservoirs. The average water age is a measure for the water quality, and as such must be kept below a certain value to ensure the quality. The daily water change controls the average water age. Therefore constraints are put on the water exchange rate in the reservoir.

The time integral of the inflow to the reservoir is a measure of the exchanged water volume. With the sign convention chosen here, the inflow to the reservoir equals $-d_n \mathbb{1}_{d_n<0} = (d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(t)) \mathbb{1}_{d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(t)>0}$. The exchanged water volume is therefore given by

$$V = \int_T \left(d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(\tau) \right) \mathbb{1}_{d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(\tau)>0} \, d\tau \tag{15}$$

The volume can as well be calculated from the water flowing out of the reservoir. The water flow out of the reservoir is $d_n \mathbb{1}_{d_n \geq 0} + d_{n+1} = -(d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(t)) \mathbb{1}_{d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(t) \leq 0} + g_{\boldsymbol{\mu}}(t)$, where d_{n+1} is the water flow to PZ2, which is predicted by $g_{\boldsymbol{\mu}}$, see Fig. 1. We assume that $d_{n+1}(t) \geq 0$ ($g_{\boldsymbol{\mu}}(t) \geq 0$) for all times, which is always true in practice. Therefore, the exchanged water volume over the time horizon T is

$$V = \int_T - \left(d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(\tau) \right) \mathbb{1}_{d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(\tau) \leq 0} + g_{\boldsymbol{\mu}}(\tau) \, d\tau \tag{16}$$

To ensure the water quality, the water exchange must be larger than $\underline{V}$. Summing (15) and (16), the following inequality constraint on the water volume is obtained

$$\underline{V} < \frac{1}{2} \int_T \left| d_1 - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(\tau) \right| + g_{\boldsymbol{\mu}}(\tau) \, d\tau \tag{17}$$

4.1 Economic MPC

We will use standard economic MPC to solve the optimization problem (14). This requires a discrete system model. The only dynamic component in the water distribution network is the elevated reservoir. On integral form the reservoir dynamic is

$$h(t) = h(t - \delta t) + \frac{1}{A} \int_{t-\delta t}^{t} d_1(\tau) - \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(\tau) - g_{\boldsymbol{\mu}}(\tau) \, d\tau$$

where it is used that $\frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(t)$ predicts the demand $\bar{d}$ of PZ1 and $g_{\boldsymbol{\mu}}(t)$ predicts demand $d_{n+1}(t)$ of PZ2. The economic MPC finds an optimal set of piece-wise constant flow values $d_1(t_0), d_1(t_0+\delta t), \cdots, d_1(t_0+M\delta t)$, where M is the prediction horizon and $M\delta t = T$. Therefore the reservoir model (14b) rewrites to

$$h(t) = h(t - \delta t) + \frac{\delta t}{A} d_1(t) + \frac{1}{A}(v_{\boldsymbol{\lambda}}(t) - v_{\boldsymbol{\mu}}(t)) \tag{18}$$

where $d_1(\tau)$ is constant on $\tau \in]t - \delta t;\ t]$ and

$$v_{\boldsymbol{\lambda}}(t) = \int_{t-\delta t}^{t} \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}}(\tau) \, d\tau \qquad v_{\boldsymbol{\mu}}(t) = \int_{t-\delta t}^{t} g_{\boldsymbol{\mu}}(\tau) \, d\tau.$$

Now, define vectors $\mathbf{h} \in \mathbb{R}^M$, $\mathbf{d}_1 \in \mathbb{R}^M$, $\mathbf{v}_{\boldsymbol{\lambda}} \in \mathbb{R}^M$, and $\mathbf{v}_{\boldsymbol{\mu}} \in \mathbb{R}^M$ where $[\mathbf{v}_{\boldsymbol{\lambda}}]_i = v_{\boldsymbol{\lambda}}(t_i)$ and $[\mathbf{v}_{\boldsymbol{\mu}}]_i = v_{\boldsymbol{\mu}}(t_i)$ are the demand flow predictions for the sample times t_i; $i = 1, \cdots, M$. The dynamic system in (18) is a pure integrator, which means that

$$\mathbf{h} = \mathbf{1}h(t_0) + \lambda_0 \mathbf{G}(\delta t \mathbf{d}_1 - \mathbf{v}_{\boldsymbol{\lambda}} - \mathbf{v}_{\boldsymbol{\mu}}) \tag{19}$$

where $\mathbf{G} \in \mathbb{R}^{M \times M}$ is a lower triangular matrix with 1 in the lower triangle and 1 on the diagonal.

The supply pressure p_1 is described by (14c). To put the expression on vector form we stack the pressure for times $t_1, t_2, \cdots, t_M$ such that $[\mathbf{p}_1]_i = p_1(t_i)$. Thus (14c) implies

$$\mathbf{p}_1 = \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{d}_1) + \alpha \mathbf{h} \tag{20}$$

where $[\mathbf{f}_{\boldsymbol{\theta}}(\mathbf{d}_1)]_i = f_{\boldsymbol{\theta}}(d_1(t_i), t_i)$ and $f_{\boldsymbol{\theta}}(d, t)$ is given by (12).

Let $\mathbf{e}_M \in \mathbb{R}^M$ be a unit vector with 1 at the last entry and zero on all other entries then $h(t_M) = \mathbf{e}_M^T \mathbf{h}$, which implies that

$$h(t_0 + T) = \mathbf{e}_M^T \left(\mathbf{1}h(t_0) + \lambda_0 \mathbf{G}(\delta t \mathbf{d}_1 - \mathbf{v}_{\boldsymbol{\lambda}} - \mathbf{v}_{\boldsymbol{\mu}})\right)$$

which again implies

$$h(t_0 + T) - h(t_0) = \lambda_0 \mathbf{1}^T(\delta t \mathbf{d}_1 - \mathbf{v}_{\boldsymbol{\lambda}} - \mathbf{v}_{\boldsymbol{\mu}}) \tag{21}$$

since $\mathbf{e}_M^T \mathbf{G} = \mathbf{1}^T$.

This shows that in the case where $h(t_0+T)-h(t_0) = 0$ the sum of the flows into the reservoir $\delta t \mathbf{d}_1 - \mathbf{v}_{\boldsymbol{\lambda}} - \mathbf{v}_{\boldsymbol{\mu}}$ must equal zero, which makes perfectly sense from a physical point of view.

Using (21), the discrete optimization problem becomes

$$\min_{\mathbf{d}_1} \mathbf{d}_1^T \mathbf{C}(\mathbf{p}_1 - \mathbf{1}p_0) + (\mathbf{p}_1 - \mathbf{1}p_0)^T \mathbf{C}\mathbf{d}_1 + \kappa\lambda_0^2(\delta t \mathbf{d}_1 - \mathbf{v}_{\boldsymbol{\lambda}} - \mathbf{v}_{\boldsymbol{\mu}})^T \mathbf{1}\mathbf{1}^T(\delta t \mathbf{d}_1 - \mathbf{v}_{\boldsymbol{\lambda}} - \mathbf{v}_{\boldsymbol{\mu}}) \tag{22}$$

where $\mathbf{C}$ is a diagonal cost matrix with the costs $c(t_i)$ on the diagonal. Inserting (20) and (19) in (22) the optimization problem simplifies to

$$\min_{\mathbf{d}_1} \left(\mathbf{d}_1^T \mathbf{C}\mathbf{f}_{\boldsymbol{\theta}}(\mathbf{d}_1) + \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{d}_1)^T \mathbf{C}\mathbf{d}_1 + \mathbf{d}_1^T \mathbf{Q}\mathbf{d}_1 + \mathbf{b}^T\mathbf{d}_1\right) \tag{23a}$$

where constant terms are left out and

$$\mathbf{Q} = \alpha\lambda_0(\mathbf{C}\mathbf{G} + \mathbf{G}^T\mathbf{C}) + \kappa\lambda_0^2\delta t^2 \mathbf{1}\mathbf{1}^T$$
$$\mathbf{b} = 2(\alpha h(t_0) - p_0)\mathbf{C}\mathbf{1} - 2\lambda_0(\alpha\mathbf{C}\mathbf{G} + \kappa\delta t\lambda_0\mathbf{1}\mathbf{1}^T)(\mathbf{v}_{\boldsymbol{\lambda}} + \mathbf{v}_{\boldsymbol{\mu}})$$

from which it is evident that $\mathbf{Q} = \mathbf{Q}^T > 0$. The optimization problem (23a) is subject to the following constraints

$$\underline{h}\mathbf{1} < \mathbf{h} < \overline{h}\mathbf{1} \quad , \quad \underline{d}\mathbf{1} < \mathbf{d}_1 < \overline{d}\mathbf{1} \tag{23b}$$

and the nonlinear water quality constraint

$$\underline{V} < \frac{1}{2}\mathbf{1}^T\left(|\delta t \mathbf{d}_1 - \mathbf{v}_{\boldsymbol{\lambda}}| + \mathbf{v}_{\boldsymbol{\mu}}\right) \tag{23c}$$

where $|\mathbf{x}|$ denotes the vector consisting of absolute values of the entries in $\mathbf{x}$. Furthermore, in (23c) we have used the approximation $\delta t \frac{1}{\lambda_0} g_{\boldsymbol{\lambda}} \approx v_{\boldsymbol{\lambda}}$.

5. NUMERICAL EXPERIMENT

The MPC control developed in Section 4 is tested by a numerical experiment where a model of a pipe network simulating PZ1 in Fig. 1 is used. Fig. 4 presents a sketch of this network, which has 10 nodes $n2$ to $n11$ with a demand flow, one supply node $n1$, and a node $n12$ that connects the elevated reservoir to the network. A geodetic level h_i is related to each of the node. The edges of the network is denoted $e1$ to $e11$. Each of the edges has a flow resistance r_i. The elevated reservoir has a cross-sectional area $A = 400$ m^2 and is lifted 30 meter above node $n12$ (since $n = 12$ in this case) hence $h_0 = 30$ m. In addition to the consumer demands of PZ1, there is a second consumer demand to PZ2 of Fig. 1. This demand is not affecting the network of PZ1 only the reservoir.

All the consumer demands are modelled by a nominal demand pattern added with individual stochastic variation in the form of a noise source. The nominal demand pattern is the same for all the consumer demands at nodes 2 to 11 in PZ1. The demand flow out of each of these nodes is shown in Fig. 5. The red curve is the nominal demand flow and the blue curve is the nominal flow added with noise which models natural variation in the demand. The demand flow to PZ2 is similar to the flow pattern in Fig. 5 but scaled to have a maximum flow around 18 m^3/h.

The model identification and the control are tested by simulating the network over period of 10 days. The first two days, the flow d_1 is controlled by standard on/off control based on the reservoir level. The parameters $\boldsymbol{\lambda}$, λ_0, $\boldsymbol{\mu}$, and $\boldsymbol{\theta}$ of the reduced network model are identified based on data from these two days. After the reduced model is identified, the predictive controller uses the model for calculating the optimal flow profile using the interior point algorithm in Matlabs *fmincon* solver. The algorithm uses measurements of the reservoir level h, the pressure at the supply node p_1, the flow at the supply node d_1,and the flow into PZ2 d_{n+1}.

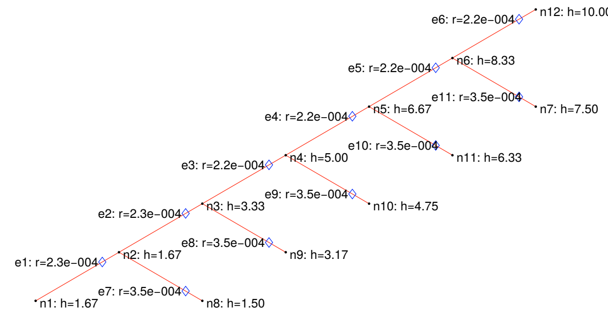

Fig. 4. Network diagram over the piping in PZ1. Node 1 is connected to pumping station and node 12 connects the network to the elevated reservoir. Note that the structure in Fig. 2 can be obtained by superimposing demands in $n2-n8$, $n3-n9$, $n4-n10$, $n5-n11$ and $n6-n7$.

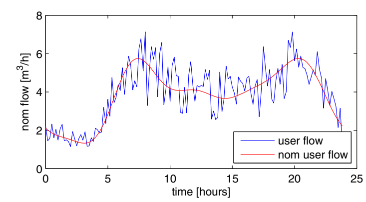

Fig. 5. End user flow profile for the user demand nodes. The red curve is the nominal user demand and the blue curve is the nominal flow added with statistical variation modelling variations in user demands.

The results of the parameter identification described in Section 3 are presented in Fig. 6. The top plot shows the real and estimated demand flows of PZ1. The middle plot shows the real and estimated demand flow to PZ2, and finally the lower plot shows the real and estimated pressure at the supply node (node 1 on the network in Fig. 4). The first day, there are no estimates as the data needed is not available before the end of the day. After the first day the real and estimated signals align closely, which exemplifies the usability of the parameter identification algorithm described in Section 3.

The numerical results obtained with the predictive control is presented in Fig. 7. The top plot shows the level change in the elevated reservoir together with the maximum and minimum level requirements, the middle plot shows the price signal, which here is a repeating signal that is high during daytime and low at night. The repeating behaviour is not necessary for the algorithm to work, but it enables a better understanding of the behaviour of the algorithm. Finally, the lower plot shows the demand flows of PZ1 (blue curve), the demand flow of PZ2 (green curve) and the supply flow calculated by the predictive control algorithm

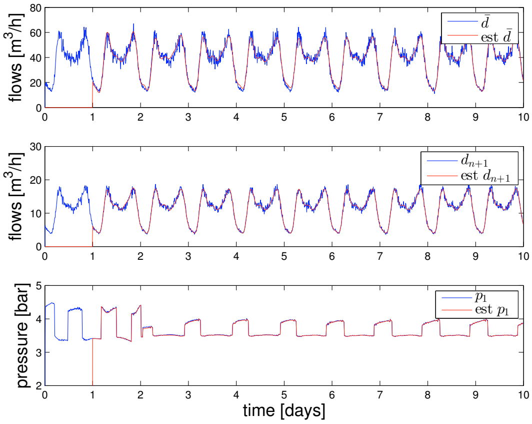

Fig. 6. Results of the model identification. The top plot is the real and the estimated user demand for PZ1, the middle plot is the real and estimated user demand of PZ2, and the lower plot is the real and estimated pressure and node 1.

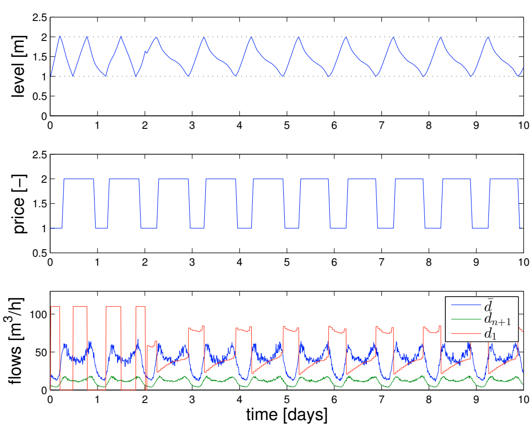

Fig. 7. Result of the predictive control. The top plot shows that level in the elevated reservoir, the middle plot shows the power price, and the lower plots show the demand flows in PZ1 and 2, and the supply flow obtained by the optimal predictive controller.

(red curve).
The results presented in Fig. 7 show that the predictive control algorithm is activated after two days, at which time it is expected that the reduced network model parameters are valid. From the level signal it is evident that the reservoir is filled when the energy price is low and empties at the time where the energy price is high, which seems plausible from an cost optimal point of view. Also, note that the flow d_1 is increasing during the emptying phase. This is because the power consumption is a function of the pressure at node 1, which is a function of the flow and the reservoir level. Therefore, the flow is increased with decreased reservoir level, which results in almost constant supply pressure p_1 (see the lower plot of Fig. 6).

6. CONCLUSION

This paper studies Model Predictive Control (MPC) of a network configuration found in many European water distribution networks. The paper utilizes a reduced network model that enables automatic identification of the network behaviour. An economical MPC control approach is developed based on the model which takes non-linearities in the hydraulic network into account. The approach is tested on a numerical simulation of a small network. The tests show that the system is able to optimize the pump operation after two days, without the need for any preknowledge about the network and reservoir, making the control system plug-and-play commissionable.
Future work include methods for controlling multiple pumping stations by utilizing distributed MPC and for smoothing the supply flow d_1 by the use of continuous control methods.

REFERENCES

Alcocer-Yamanaka, V., Tzatchkov, V., and Arreguin-Cortes, F. (2012). Modeling of drinking water distribution networks using stochastic demand. *Water Resour Manage (2012) 26:17791792*, 26, 17791792.

Aquacraft, I. (2011). Embedded energy in water studies study 3: End-use water demand profiles. CALMAC Study ID CPU0052. Prepared for California Public Utilities Commission; Apr, 2011.

Baunsgaard, K., Ravn, O., Kallesøe, C., and Poulsen, N. (2016). Mpc control of water supply networks.

Kallesøe, C.S., Aarestrup, J., and Rokkjær, K. (2011). Energy optimization for booster sets. World Pumps. Technical note.

Kallesøe, C., Jensen, T., and Wisniewski., R. (2015). Adaptive reference control for pressure management in water networks.

Leirens, S., Zamora, C., Negenborn, R., and Schutter, B.D. (2010). Coordination in urban water supply networks using distributed model predictive control. In *American Control Conference, Marriott Waterfront, Baltimore, MD, USA*.

Ljung, L. (1999). *System identification theory for the user.* Prentice-Hall.

Madsen, H. (2008). *Time Series Analysis.* Chapman & Hall/CRC.

Maschler, T. and Savic, D. (1999). Simplification of water supply network models through linearisation, centre for water systems. *Report No.99/01, School of Engineering, University of Exeter, Exeter, United Kingdom.*

Ocampo-Martinez, C., Puig, V., and Bovo, S. (2011). Decentralised mpc based on a graph partitioning approach applied to the barcelona drinking water networkl. In *18th IFAC World Congress, Milano, Italy.*

Ocampo-Martinez, C., Puig, V., Cembrano, G., and Quevedo, J. (2013). Application of predictive control strategies to the management of complex networks in the urban water cycle. *IEEE Control Systems*, 33(1), 15–41.

Paluszczyszyn, D., Skworcow, P., and Ulanicki, B. (2013). Online simplification of water distribution network models for optimal scheduling. *Journal of Hydroinformatics*, 15(3), 652665.

Stoustrup, J. (2009). Plug & play control: Control technology towards new challenges. *European Journal of Control*, 15(3-4), 311–330.

Sun, C., Puig, V., and Cembrano, G. (2014). Combining csp and mpc for the operational control of water networks application to the richmond case study. In *19th IFAC World Congress, Cape Town, South Africa.*